

Introduction to expanding ergodic optimization

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1 Introduction

These lecture notes grew out of a graduate course on ergodic optimization given by the author at the University of Campinas. Obviously some background in ergodic theory is required to follow the text. Moreover, these notes are by no means meant to be exhaustive. As a matter of fact, we focus mostly on the interpretation of ergodic optimal problems as questions of variational dynamics (see, for instance, [30, 37, 38, 55]), in a comparable way to the Aubry-Mather theory for Lagrangian systems. The reader shall be conscious that other points of view are also useful in ergodic optimization, like the one based on properties of Sturmian measures and its generalizations (see, for example, [14, 21, 48]).

Ergodic optimization is a theoretical branch primarily concerned with the study of the so-called optimizing probability measures. The goal of this introductory monograph is hence twofold. One objective is to present and