Discrete weak-KAM methods for stationary uniquely ergodic setting

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Abstract

The Frenkel-Kontorova model describes how an infinite chain of atoms minimizes the total energy of the system when the energy takes into account the interaction of nearest neighbors as well as the interaction with an exterior environment. An almost-periodic environment leads to consider a family of interaction energies which is stationary with respect to a minimal topological dynamical system. We introduce, in this context, the notion of calibrated configuration (stronger than the standard minimizing condition) and, for continuous superlinear interaction energies, we prove its existence for some environment of the dynamical system. Furthermore, in one dimension, we give sufficient conditions on the family of interaction energies to ensure the existence of calibrated configurations for any environment when the underlying dynamics is uniquely ergodic. The main mathematical tools for this study are developed in the frameworks of discrete weak KAM theory, Aubry-Mather theory and spaces of Delone sets.

Keywords: almost-periodic environment, Aubry-Mather theory, calibrated configuration, Delone set, Frenkel-Kontorova model, Mañé potential, Mather set, minimizing holonomic probability, weak KAM theory

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